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International Journal of Heat and Mass Transfer 49 (2006) 5081–5085
International Journal of HEAT and MASS TRANSFER
www.elsevier.com/locate/ijhmt

Technical Note

Transient thermal behavior of porous media under oscillating flow condition

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Received 10 April 2006; received in revised form 10 April 2006
Available online 17 August 2006

Abstract

An analytical characterization of the heat transfer in an oscillating flow through a porous medium is presented in this work. Based on a two-equation model, two important dimensionless parameters are identified as the ratio of the thermal capacities between the solid and fluid phases and the ratio of the interstitial heat conductance between the phases to the fluid thermal capacity. The analytic solutions are obtained for both the fluid and solid temperature variations, and the heat transfer characteristics between the phases are classified into four regimes. In addition, a criterion for the validity of the local thermal equilibrium is suggested in a simple form as the ratio of the two time scales intrinsically involved in any transient heat transfer in porous media, namely the time scale relevant to the thermal inertia of porous media and the time scale pertinent to the transient variation of the boundary condition.

Keywords: Transient thermal behavior; Porous media; Oscillating flow; Thermal equilibrium

1. Introduction

Porous media have been widely used in industry as an effective means of transporting and storing thermal energy. Common examples of the industrial applications include thermal regenerators of the Stirling cycles, rotary regenerative heat exchangers, and temporary energy storage units. In such applications, the transient characteristics of the porous media are of importance since the porous media absorb and release thermal energy periodically [1].

One of the early investigations on the transient heat transfer in porous media was performed by Riaz [2]. He analyzed the unsteady response of thermal storage system to a step change in the inlet air temperature. Spiga and Spiga [3] analytically obtained the dynamic response of the thermal storage system to an arbitrary time-varying inlet temperature. Recently, the case where the flow oscillates through a porous medium were investigated by Muralidhar and Suzuki [4], and Klein and Eigenberger [5]. They analyzed numerically or theoretically the periodic heat transfer in porous media for the analysis of thermal regenerators. Most of the studies mentioned above dealt with the problems by means of numerical integrations or complicated series solutions. These means, however, are not very suitable for deduction of fundamental aspects of periodic heat transfer in porous media underlying the apparent complex phenomena.

The main objective of this study is to analyze theoretically the transient heat transfer in porous media under oscillating flow condition. Exact solutions are obtained for both the fluid and solid temperature variations, and the transient thermal characteristics are investigated theoretically based on the solutions. Additionally, the temperature difference between the phases is examined and a simple

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0017-9310/$ - see front matter
doi:10.1016/j.ijheatmasstransfer.2006.05.036

Nomenclature

a	interfacial area per unit volume of porous media (m^{-1})
C_p	isobaric heat capacity ($J\,kg^{-1}\,K^{-1}$)
g	complex amplitude of oscillating temperature
h	interstitial heat transfer coefficient ($W\,m^{-2}\,K^{-1}$)
K	thermal capacity ratio defined in Eq. (7)
k	thermal conductivity ($W\,m^{-1}\,K^{-1}$)
L	length of porous media in flow direction (m)
L_s	oscillation distance of flow (m)
S	ratio of interstitial thermal conductance to fluid thermal capacity
T	temperature (K)
T_0	time-averaged temperature (K)
t	time (s)
t_o	time scale of oscillating flow (s)
t_p	characteristic time of porous media (s)
u	fluid velocity ($m\,s^{-1}$)
x	longitudinal coordinate (m)

Greek symbols

θ	non-dimensional temperature
ε	porosity
γ	gradient of the linear temperature distribution ($K\,m^{-1}$)
τ	non-dimensional time
ρ	density ($kg\,m^{-3}$)
ω	frequency (s^{-1})

Subscripts

eff	effective value
f	fluid
s	solid
0	reference point

criterion is prescribed for the validity of the local thermal equilibrium.

2. Theoretical analysis

Fig. 1 shows an infinitely large slab of a porous medium with a thickness of L. The fluid at each side of the slab is maintained at high and low temperatures, respectively. The flow oscillates back and forth through the porous slab and transfers heat from the hot end to the cold end of the slab. This situation commonly happens in the regenerators of the ventilation systems and in the thermal regenerative engines or coolers. The energy equation for each phase can be written as [3]:

Fluid phase:

$$\varepsilon(\rho C_p)_f \frac{\partial T_f}{\partial t} + \varepsilon(\rho C_p)_f u \frac{\partial T_f}{\partial x} = k_{f,eff} \frac{\partial^2 T_f}{\partial x^2} + ha(T_s - T_f). \quad (1)$$

Solid phase:

$$(1-\varepsilon)(\rho C_p)_s \frac{\partial T_s}{\partial t} = k_{s,eff} \frac{\partial^2 T_s}{\partial x^2} + ha(T_f - T_s). \quad (2)$$

The oscillating velocity in the above equation is expressed as

$$u = \frac{L_s \omega}{2} \cos(\omega t), \quad (3)$$

where L_s is the swept distance and ω is the frequency of the oscillating flow.

When the representative pore diameter of the porous medium is sufficiently small compared to the thickness of the slab as is often the case, it is well known that the entrance region near each end of the porous slab is negligibly small compared to the thickness of the slab [6,7]. Neglecting the entrance region, the temperature within the slab has been found to have a linear distribution [4,5,8]. This finding implies a negligible contribution of

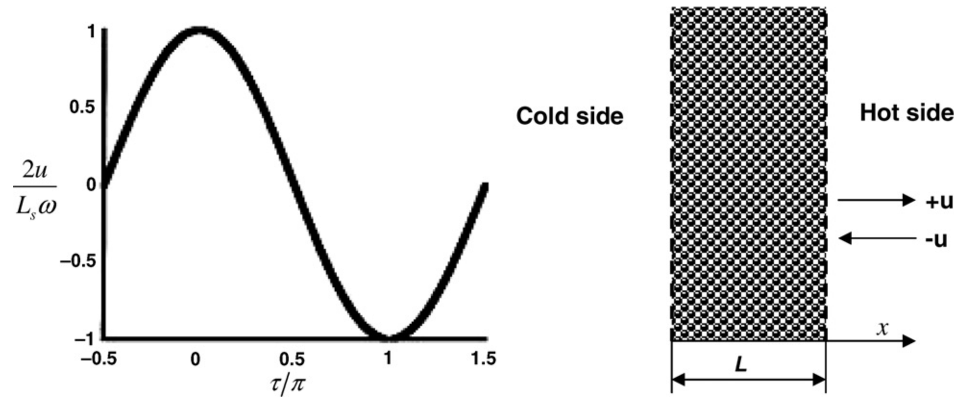

Fig. 1. Schematics of the model.

the fluid dispersion or the solid conduction to the local energy transfer in porous media under oscillating flow condition.

Considering the linear temperature distribution, a non-dimensional temperature can be defined as

$$\theta = \frac{T - T_0(x)}{L_s\gamma/2}, \quad T_0(x) = \gamma(x - x_0). \tag{4}$$

In the above equation, T_0 implies the time-averaged temperature within the slab, γ is the gradient of the linear temperature distribution, and x_0 is the point where T_0 is set to be 0. Considering the linear temperature distribution and using the non-dimensional temperature, Eqs. (1) and (2) are simplified and rendered dimensionless as

$$\frac{d\theta_f}{d\tau} + \cos(\tau) = S(\theta_s - \theta_f), \tag{5}$$

$$K\frac{d\theta_s}{d\tau} = S(\theta_f - \theta_s), \tag{6}$$

where two non-dimensional parameters, S and K, are defined as

$$S = \frac{ha}{\varepsilon(\rho C_p)_f\omega}, \quad K = \frac{(1-\varepsilon)(\rho C_p)_s}{\varepsilon(\rho C_p)_f}. \tag{7}$$

The parameter S is the ratio of the interstitial heat conductance between the phases to the fluid thermal capacity. The parameter K represents the ratio of the thermal capacities between the solid and fluid phases. Conceptually, S and K imply two important aspects of the porous medium as a regenerator, i.e., how fast or how large an amount of heat can be transferred from the fluid phase to the solid phase.

The exact solution to the equations can be sought as

$$\theta(\tau) = \mathrm{Real}\lfloor g \cdot e^{i\tau}\rfloor, \tag{8}$$

where g is a complex number containing information about the amplitude and the phase angle of the temperature variation. Substituting Eq. (8) into Eqs. (5) and (6), and solving the subsequent algebraic equations yields

$$g_f = \frac{S + iK}{K - iS(1+K)}, \tag{9}$$

$$g_s = \frac{S}{K - iS(1+K)}. \tag{10}$$

The timewise temperature variation of each phase can be obtained by substituting Eqs. (9) and (10) into Eq. (8).

3. Results and discussion

The temperature variation of the fluid and solid phases for a range of parameters, K and S, is shown in Fig. 2 in which the time period of one cycle is selected as $-\pi/2 \leqslant \tau \leqslant 3\pi/2$. Referring to Eq. (3), the left half of each figure represents the cold blow period and the right half represents the hot blow period.

When K is relatively large and S is small as in Fig. 2(a), the temperature variation of the fluid phase is very large whereas that of the solid phase is negligibly small. In a

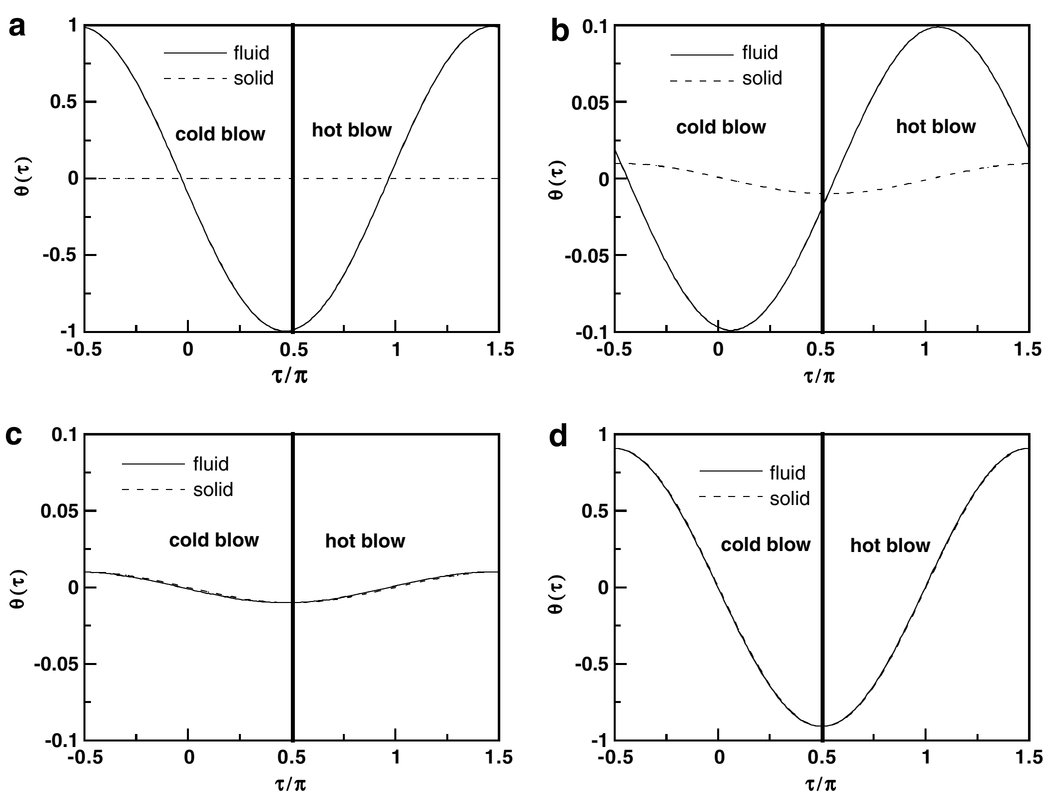

Fig. 2. Temperature variations for four different cases: (a) $K = 100$, $S = 0.1$; (b) K = 100, S = 10; (c) $K = 100$, $S = 1000$ and (d) $K = 0.1$, $S = 10$.

regenerator for thermal recovery, a large fluid temperature variation can be interpreted as poor thermal regeneration. As S increases for a fixed value of K, the temperature variation of the fluid phase becomes smaller and that of the solid phase becomes larger as shown in Fig. 2(b). The increase in the amplitude of the solid temperature variation represents the increase in the amount of heat absorbed and released by the solid phase. For a further increase of S even larger than K, the solid temperature maintains almost the same while the fluid temperature oscillation decreases to have a similar temperature variation to the solid phase as shown in Fig. 2(c). In the meanwhile, if K decreases from the case in Fig. 2(c), the temperature oscillations increase for both the solid and fluid phases as shown in Fig. 2(d).

To show more clearly the effects of the parameters on the transient heat transfer, the amplitude of oscillating temperature is plotted for each phase as a function of S and K in Fig. 3. Each of Fig. 3(a) and (b) shows the existence of distinct regimes with different dependency on the parameters. From the three-dimensional plots in Fig. 3, four distinct regimes can be identified and are presented in Fig. 4. The range of the parameters in each regime is arranged in Table 1. Table 1 also summarizes the reduced governing equations and asymptotic solutions to the temperature variations in each regime. The asymptotic temperature variations were obtained by applying extreme values of the parameters, S and K, within the range of each regime to Eqs. (9) and (10). They are found the same with those directly obtainable from the reduced governing equations.

The simplified form of governing equations provides a clearer physical view of the heat transfer characteristics by identifying dominant factors in each regime. From Table 1, the energy convected by oscillating flow is found to balance with the energy stored in the solid phase in regimes II and III, whereas it does with the energy stored in the fluid phase in regimes I and IV. Since both the parameters, S and K are larger than one in regimes II and III, the convected energy can be transferred to the solid phase by the large interstitial heat conductance and also can be stored in the solid phase due to the large solid thermal capacity. If either of the parameters is less than one, the convected energy can not be stored in the solid phase.

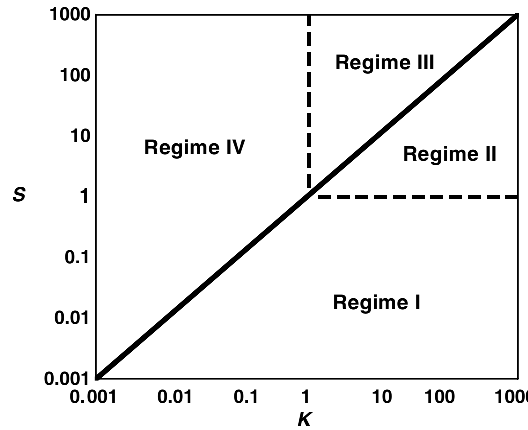

Fig. 4. Regime classification.

Table 1 also shows that the temperature of each phase is asymptotically the same in regimes III and IV, while it is not in regimes I and II. From this founding, the criterion for local thermal equilibrium can be arranged as

$$\frac{K}{S} = \frac{(1-\varepsilon)(\rho C_p)_s\omega}{ha} \ll 1. \tag{11}$$

This criterion describes that the local thermal equilibrium can be achieved when the interstitial heat conductance, S, is larger than the thermal capacity of the solid phase, K.

While the flow is steady and unidirectional, the works on the transient heat transfer subject to unsteady thermal boundary conditions are also worthy of note. Spiga and Spiga [3] investigated theoretically the dynamic response of porous media to an arbitrary time varying inlet temperature. Though they have not mentioned about the thermal equilibrium explicitly, it can be readily shown that the temperature difference between the phases diminishes in their solution when the same condition as expressed in Eq. (11) is satisfied with ω being the frequency of the periodic variation of the thermal boundary condition. It is quite

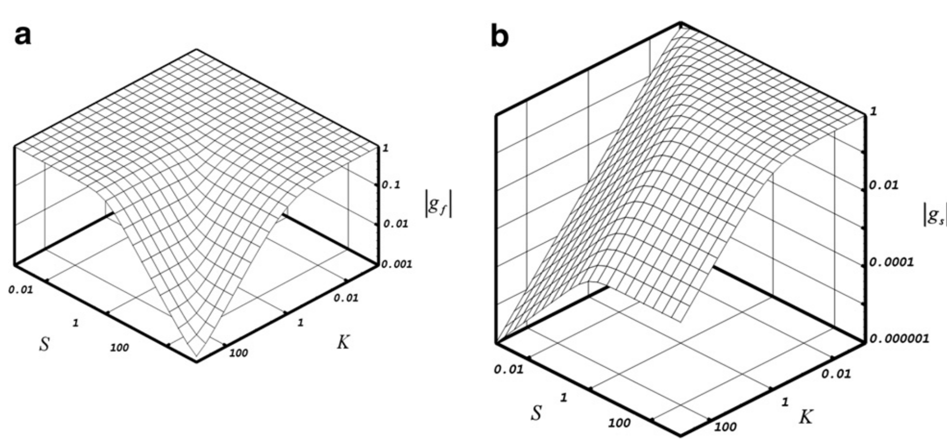

Fig. 3. Amplitudes of oscillating temperatures: (a) fluid and (b) solid.

Table 1
Reduced governing equations and asymptotic solutions within each regime

Regime	Range	Reduced governing equations	Asymptotic solutions
I	$K \gg S$ and $S \ll 1$	$\frac{d\theta_f}{d\tau} \approx -\cos\tau, \quad K\frac{d\theta_s}{d\tau} \approx S\theta_f$	$\theta_f \approx -\sin\tau, \quad \theta_s \approx \frac{S}{K}\cos\tau$
II	$K \gg S \gg 1$	$S\theta_f \approx -\cos\tau, \quad K\frac{d\theta_s}{d\tau} \approx S\theta_f$	$\theta_f \approx -\frac{1}{S}\cos\tau, \quad \theta_s \approx -\frac{1}{K}\sin\tau$
III	$S \gg K \gg 1$	$K\frac{d\theta_s}{d\tau} \approx -\cos\tau, \quad \theta_f \approx \theta_s$	$\theta_f \approx -\frac{1}{K}\sin\tau, \quad \theta_s \approx -\frac{1}{K}\sin\tau$
IV	$S \gg K$ and $K \ll 1$	$\frac{d\theta_f}{d\tau} \approx -\cos\tau, \quad \theta_f \approx \theta_s$	$\theta_f \approx -\sin\tau, \quad \theta_s \approx -\sin\tau$

interesting that the resulting criterion for the thermal equilibrium is exactly the same even though the situation of the transient heat transfer is completely different. From this finding, it might be inferred that there exists a general criterion for the thermal equilibrium in transient heat transfer in porous media independent of the detailed thermal or flow boundary conditions. In this respect, the criterion, Eq. (11) is rearranged in a different expression as

$$\frac{K}{S} = \frac{(1-\varepsilon)(\rho C_p)_s\omega}{ha} = \frac{t_p}{t_o} \ll 1, \tag{12}$$

where

$$t_o = \frac{1}{\omega} \quad \text{and} \quad t_p = \frac{(1-\varepsilon)(\rho C_p)_s}{ha}. \tag{13}$$

In the above equation, t_o is the time scale of the variation of the flow or thermal boundary condition, and t_p is the time scale concerning the thermal inertia of the porous media. Now the criterion is expressed as the ratio of the two time scales intrinsically involved in any transient heat transfer problem in porous media. The criterion describes that the local thermal equilibrium can be achieved when the characteristic time of the porous media is much shorter than the time scale concerning the variation of the boundary condition.

4. Conclusions

An analytical characterization of the transient heat transfer in porous media under the oscillating flow condition is presented in this work. Based on a two-equation model, two important dimensionless parameters are identified as the ratio of the thermal capacities between the solid and fluid phases and the ratio of the interstitial heat conductance between the phases to the fluid thermal capacity. The analytic solutions are obtained for both the fluid and solid temperature variations, and the heat transfer characteristics between phases are classified into four regimes.

In addition, a criterion for the validity of the local thermal equilibrium is suggested in a simple form as the ratio of the two time scales intrinsically involved in any transient heat transfer in porous media, namely the time scale relevant to the thermal inertia of porous media and the time scale pertinent to the transient variation of the boundary condition. The local thermal equilibrium can be achieved when the characteristic time of the porous media is much shorter than the time scale concerning the variation of the boundary condition.

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